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Microscale temperature and velocity spectra in the atmospheric boundary layer

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High-frequency fluctuations in temperature and velocity were measured at a height of 2 m above a harvested, nearly level field of rye grass. Conditions were both stably and unstably stratified. Reynolds numbers ranged from 370000 to 740000. Measurements of velocity were made with a hot-wire anemometer and measurements of temperature with a platinum resistance element which had a diameter of 0·5 μm and a length of 1 mm. Thirteen runs ranging in length from 78 to 238 s were analysed.

Spectra of velocity fluctuations are consistent with previously reported universal forms. Spectra of temperature, however, exhibit an increase in slope with increasing wavenumber as the maximum in the one-dimensional dissipation spectrum is approached. The peak of the one-dimensional dissipation spectrum for temperature fluctuations occurs at a higher wavenumber than that of simultaneous spectra of the dissipation of velocity fluctuations. It is suggested that the change in slope of the temperature spectra and the dissimilarity between temperature and velocity spectra may be due to spatial dissimilarity in the dissipation of temperature and velocity fluctuations. The temperature spectra are compared with a theoretical prediction for fluids with large Prandtl number, due to Batchelor (1959). Even though air has a Prandtl number of 0·7, the observations are in qualitative agreement with predictions of the theory. The non-dimensional wavenumber at which the increase in slope occurs is about 0·02, in good agreement with observations in the ocean reported by Grant *et al.* (1968).

For the two runs for which the stratification was stable, the normalized spectra of the temperature derivative fall on average slightly below the mean of the spectra of the remaining runs in the range in which the slope is approximately one-third. Hence the Reynolds number may not have always been sufficiently high to satisfy completely the conditions for an inertial subrange.

Universal inertial-subrange constants were directly evaluated from one-dimensional dissipation spectra and found to be 0·54 and 1·00 for velocity and temperature, respectively. The constant for velocity is consistent with previously reported values, while the value for temperature differs from some of the previous direct estimates but is only 20 % greater than the mean of the indirect estimates. This discrepancy may be explained by the neglect in the indirect estimates of the divergence terms in the conservation equation for the variance of temperature fluctuations. There is weak evidence that the one-dimensional constant, and hence the temperature spectra, may depend upon the turbulence Reynolds number, which varied from 1200 to 4300 in the observations reported.

1. Introduction

Investigations of high-frequency temperature and velocity fluctuations in flows with high Reynolds number have been reviewed by Monin & Yaglom (1975). The observational investigation reported below is very similar to that of Boston & Burling (1972). Therefore only the previous theory and observations which are essential to the logical development of the present paper are reviewed.

Investigations of high-frequency velocity fluctuations in flows with high Reynolds number have established, within an uncertainty of about $\pm 10\%$ (Monin & Yaglom 1975, p. 485), the value of the Kolmogorov constant α for the $-\frac{5}{3}$-region of the one-dimensional velocity spectrum given by

$$\phi_u(k) = \alpha\epsilon^{\frac{2}{3}}k^{-\frac{5}{3}} \tag{1}$$

(Kolmogorov 1941), where ϵ is the mean rate of dissipation of turbulent kinetic energy per unit mass and k is the downstream radian wavenumber. Φ_u is defined such that

$$\int_0^\infty \phi_u(k)\,dk = \overline{u^2}, \tag{2}$$

where $\overline{u^2}$ is the variance of the downstream velocity fluctuations. If the turbulence is locally isotropic for scales at which dissipation is significant, ϵ is given by

$$\epsilon = 15\nu\overline{(\partial u/\partial x)^2}. \tag{3}$$

By use of (2), ϵ can be related to the derivative spectrum by

$$\epsilon = 15\nu\int_0^\infty \phi_{u'}(k)\,dk, \tag{4}$$

where x is the downstream space co-ordinate, u' is the downstream spatial derivative of u, ν is the kinematic viscosity and the overbar implies either a space or a time average. $\phi_{u'}(k)$ is referred to as the one-dimensional dissipation spectrum or, in this paper, the dissipation spectrum. The spatial derivative may be converted to a time derivative by use of Taylor's hypothesis (Monin & Yaglom 1975, p. 449).

Observations (e.g. Boston & Burling 1972) have also supported the prediction by Kolmogorov (1941) that for sufficiently high Reynolds numbers the one-dimensional velocity spectrum for the locally isotropic scales will have the form

$$\phi_u(k) = (\epsilon\nu^5)^{\frac{1}{4}}\,F(k/k_s), \tag{5}$$

where k_s ($= (\epsilon/\nu^3)^{\frac{1}{4}}$) is the Kolmogorov wavenumber and $F(k/k_s)$ is a universal function. A similar relation follows for the dissipation spectrum:

$$k^2\phi_u(k) = k_s^2(\epsilon\nu^5)^{\frac{1}{4}}\,(k/k_s)^2\,F(k/k_s). \tag{6}$$

Kolmogorov (1962) revised the classical theory (Kolmogorov 1941) to account for the variability of ϵ. The revised theory suggests that the shape of the velocity spectrum is not universal, but depends on the turbulence Reynolds number. However, theory (Yaglom 1966; Wyngaard & Tennekes 1970) suggests that the dependence is weak and difficult to detect observationally.

Arguments of the type advanced by Kolmogorov (1941) for velocity fluctuations in high Reynolds number flows have also been advanced for fluctuations of scalars (Oboukhov 1949; Corrsin 1951; Batchelor 1959). These arguments lead to relations analogous to those for velocity. For a sufficiently high Reynolds number, there is predicted a $-\frac{5}{3}$-region of the one-dimensional spectrum of temperature given by

$$\phi_\theta(k) = \beta \epsilon^{-\frac{1}{3}} \epsilon_\theta k^{-\frac{5}{3}}, \tag{7}$$

where β is a universal constant while ϵ_θ is the mean rate of dissipation of turbulent temperature fluctuations ($\frac{1}{2}\overline{\theta^2}$) and is given by

$$\epsilon_\theta = 3D_\theta \overline{(\partial\theta/\partial x)^2} \tag{8}$$

$$= 3D_\theta \int_0^\infty \phi_{\theta'}(k)\, dk, \tag{9}$$

where D_θ is the thermal diffusivity and θ' is the downstream spatial derivative of θ. $\phi_\theta(k)$ is defined such that

$$\int_0^\infty \phi_\theta(k)\, dk = \overline{\theta^2}, \tag{10}$$

where $\overline{\theta^2}$ is the variance of turbulent temperature fluctuations in the direction of the mean flow. We also have the prediction that, for the range of scales which are locally isotropic,

$$\phi_\theta(k) = \epsilon_\theta (\nu^5/\epsilon^3)^{\frac{1}{4}} H(Pr, k/k_s), \tag{11}$$

$$k^2\phi_\theta(k) = k_s^2 \epsilon_\theta (\nu^5/\epsilon^3)^{\frac{1}{4}} (k/k_s)^2 H(Pr, k/k_s), \tag{12}$$

where H is a universal function and Pr is the Prandtl number ν/D_θ.

It should be noted that the definitions of α and β vary among authors depending on whether a factor of $\frac{1}{2}$ is included on the right side of (10) or whether one defines ϵ_θ as the dissipation of $\overline{\theta^2}$ or $\frac{1}{2}\overline{\theta^2}$. Variations also occur owing to the definition of k as either a radian or non-radian wavenumber. Paquin & Pond (1971) discuss these variations. The convention adopted here is also that of Monin & Yaglom (1975).

The universal constant β defined in (7) is not nearly so well determined as the corresponding constant for velocity spectra. Only a few estimates of β have been reported which were determined by measurement of temperature and velocity fluctuations to small enough scales to estimate ϵ and ϵ_θ by use of (4) and (9). Gibson, Stegen & Williams (1970) report $\beta = 2{\cdot}3$ from measurements over the sea while Boston & Burling (1972) report $\beta = 1{\cdot}6$ from measurements over a mudflat. Clay (1973) reports values of β ranging from 0·8 to 1·3 determined from laboratory measurements in air, water and mercury and a value of β equal to 1·6 determined from measurements in the atmosphere.

Gibson & Schwarz (1963) estimated $\beta = 0{\cdot}7$ from laboratory measurements in air and water. The dissipation rates of temperature and velocity fluctuations were determined from the decay of the fluctuations downstream from a grid.

There have been many indirect experimental estimates of β. The indirect determinations are usually based on estimating ϵ_θ as a residual in the conservation equation for turbulent temperature fluctuations (e.g. Wyngaard & Coté 1971), often with neglect of the divergence terms. Another indirect method is based on relating β to the second-and third-order moments of the temperature and velocity fluctuations in the inertial subrange (Paquin & Pond 1971). Monin & Yaglom (1975) conclude from a review of the indirect estimates that β is near 0·75.

Batchelor (1959) suggested a universal, theoretical temperature spectrum valid for fluids with large Prandtl number and for wavenumbers greater than k_*, where k_* is defined as the high-wavenumber termination of the inertial subrange. The one-dimensional form of this spectrum was given by Gibson & Schwarz (1963):

$$\phi_\theta(k) = 4\pi^{\frac{1}{2}} q^{\frac{3}{2}} D_\theta^{\frac{1}{2}} (\nu/\epsilon)^{\frac{3}{4}} \epsilon_\theta \left[\frac{N(B)}{B} - \int_B^\infty N(y)\,dy \right], \quad k > k_*, \tag{13}$$

where $B = (2q)^{\frac{1}{2}} k/k_B$, k_B = Batchelor's characteristic wavenumber $= (\epsilon/\nu D_\theta^2)^{\frac{1}{4}}$, $N(B)$ = normal probability density function $= (2\pi)^{-\frac{1}{2}} \exp(-\frac{1}{2}B^2)$ and q = universal constant related to the effective average value γ of the least principal rate of strain $= -1/\gamma(\epsilon/\nu)^{\frac{1}{2}}$. Equation (7) is valid for wavenumbers less than k_*. For values of k which are sufficiently small, (13) reduces approximately to

$$\phi_\theta(k) = 2q\nu^{\frac{1}{2}}\epsilon^{-\frac{1}{2}}\epsilon_\theta k^{-1}, \tag{14}$$

which is called the viscous–convective subrange. For increasing wavenumber the viscous–convective subrange is followed by an exponential decrease of $\phi(k)$. From (7) and (14) one may obtain the relation

$$\beta = 2q(k_*/k_s)^{\frac{2}{3}}, \tag{15}$$

where β, q and k_*/k_s are all universal.

Batchelor suggested that q would be about 2 and that k_*/k_s would be of order one. Gibson & Schwarz (1963) found k_*/k_s to be about 0·1 and $q = 2$ from laboratory measurements. Gibson, Lyon & Hirschsohn (1970) found values of 0·03 and 0·04 for k_*/k_s but did not estimate β or q. Grant *et al.* (1968) found k_*/k_s to be equal to 0·024 and q to about 4 from measurements in the open ocean and in a tidal channel. Batchelor's (1959) theory was not intended to apply to measurements in air ($Pr = 0{\cdot}7$). However, E. A. Novikov (see Monin & Yaglom 1975, p. 441) has suggested that the theory may be valid in fluids having Pr of order one.

Van Atta (1971, 1973) investigated theoretically the effect of intermittency on the temperature spectrum and structure function in the inertial subrange. His approach is similar to that of Oboukhov (1962), Kolmogorov (1962) and Gurvich & Yaglom (1967), who suggested modifications to the velocity structure function due to fluctuations in dissipation. Van Atta suggested that the temperature structure function depends on the correlation $\rho(r)$ between fluctuations in the dissipation of temperature and velocity averaged over a volume with radius r. From dimensional arguments and by assuming the joint probability of temperature and velocity dissipations to be bivariate lognormal, he arrived at an expression for the temperature structure function in the inertial subrange. His results can be shown to be equivalent to a prediction of the slope of the temperature dissipation spectrum in the inertial subrange given by

$$m = \tfrac{1}{3}(1 + \tfrac{2}{3}\mu - \rho\mu), \tag{16}$$

where μ is a constant appearing in the lognormal probability distributions for the dissipation rates, assumed the same for both velocity and temperature. For no variability in dissipation rates, $\mu = 0$ and m reduces to one-third. Measurements (see Wyngaard & Pao 1971) indicate $\mu \simeq 0{\cdot}5$. Thus

$$m \simeq \tfrac{1}{3}(\tfrac{4}{3} - \tfrac{1}{2}\rho). \tag{17}$$

As ρ decreases, m increases, taking the classical value of one-third for $\rho = \frac{2}{3}$.

Motivated by the uncertainty in the value of β and the form of the temperature spectrum at large wavenumbers, observations of temperature and velocity fluctuations were obtained over a horizontally uniform terrestrial site and from these β was determined directly by use of (4), (7) and (9). It is the purpose of this paper to report these observations and subsequent analyses.

2. Instrumentation

Determination of the viscous and thermal rates of dissipation directly from measurements of velocity and temperature in the atmospheric boundary layer imposes certain requirements on the instrumentation to be used. The maximum dimensions of the sensors must not be greater than the typical Kolmogorov microscale (about 1 mm for both temperature and velocity) and the frequency response must be sufficient to resolve the smallest scales as they are advected past the sensors by the mean flow (about 2000 Hz). The noise of the instrumentation must be small compared with typical Kolmogorov microscales for velocity and temperature ($\sim$ 3 cm/s and $\sim$ 0·005 °C respectively).

The instrumentation requirements for velocity measurements pose no difficulty since there are several commercially available hot-wire or hot-film anemometer systems which are suitable. A Thermo-Systems Model 1054 A anemometer was employed in the observations reported here. The sensing element was a tungsten wire of diameter 4 μm and length 1·25 mm.

The sensor chosen for the temperature measurements was a platinum wire with a diameter of 0·5 μm and a length of 1 mm. The sensor was constructed by soldering a silver-jacketed (Wollaston) wire to conventional hot-wire anemometer needle supports and then etching away the silver over a 1 mm section to expose the platinum. The frequency response of the sensor estimated from the relation of Sandborn (1972) is approximately 4000 Hz in still air. The sensor resistance was measured with a 100 kHz a.c. symmetrical bridge constructed especially for the purpose. A discrete differential transistor pair was employed in the first-stage amplifier to maximize the signal-to-noise ratio. The current through the sensor was 100 μA. The velocity sensitivity of the temperature sensor estimated from an expression due to Wyngaard (1971*b*) was 0·00006 °C/(cm s^{-1}). The noise of the system measured over a 10 kHz bandwidth was equivalent to 0·005 °C r.m.s. For a more detailed description, see Williams (1974).

After suitable amplification and offsets, the temperature and velocity signals were recorded in frequency-modulated form on separate tracks of a Hewlett–Packard model 3955 magnetic tape-recorder operated at a speed of 15 in./s. Also recorded were electronically differentiated temperature and velocity signals. Differentiation enhances the signal-to-noise ratio of the recording in the high-frequency part of the spectrum. The cut-off frequency of the differentiator was 2000 Hz.

3. Observations

Observations were made near Corvallis, Oregon, during the summer of 1973 over a nearly level field of rye-grass stubble. Considerable straw was scattered over the stubble. For the observed wind directions the terrain upwind of the location of the measurements was uniform to distances in excess of 1·5 km. Conditions were dry with few clouds. Mean air temperatures ranged from 36 °C during the day to 25 °C at night.

The instruments were mounted at a height of 2 m on a portable mast. The temperature and velocity sensors were mounted in the same horizontal plane, separated by about 8 cm, with the hot-wire probe aligned vertically. In addition to the wind velocity measured by the vertical hot-wire anemometer, longitudinal and vertical components were measured with an X-wire sensor. These measurements were used to derive estimates of the stress and heat flux for use in estimating stability. The wind speed and direction were also measured with a cup anemometer and vane on the same mast. The instrument mount could be rotated about a vertical axis to maintain sensor orientation into the wind. Signals from each sensor were transmitted by cable to an instrument hut housing signal-conditioning and recording equipment. The hut was located approximately 30 m downwind of the mast.

About 10 h of data on 14 magnetic tapes were recorded over a period of two days. Conditions were both unstably and stably stratified. Wind speeds ranged from 2 to 6 m/s.

4. Analysis and results

The first step in the analysis was the selection of suitable sections of the data for processing. Economic constraints limited the total amount of data analysed to about 30 min (about 30 million samples after digitizing). The computation of ϵ_θ as a function of the averaging time indicated that, because of the intermittency of the temperature derivative, records had to be at least 1 min in length to yield representative values of ϵ_θ. Strip-chart recordings of the signals were examined to select periods which were free of malfunctions of instrumentation and during which the mean wind speed and direction remained nearly constant. Sections 4 min long were selected from each of six data tapes and sections 1·3 min long were selected from each of seven other tapes for a total of 13 runs. This selection included conditions of both stable and unstable stratification.

An example of a strip-chart recording of velocity and temperature together with their derivatives is shown in figure 1. The highly intermittent nature of both velocity and temperature derivatives is evident. However, the temperature derivative appears more intermittent than velocity derivatives as was always the case. The bursts of high-frequency fluctuations in temperature and velocity are correlated. The correlation would probably have been even higher had the probes been closer together.

The sections of data selected for analysis were converted from analog to digital form by use of a hybrid computer (Electronic Associates, Inc. 540/680) at a sampling rate of 4000 samples/s. The analog signals were low-pass filtered at 2000 Hz prior to digitizing to prevent aliasing.

Spectra were computed by use of a general purpose digital computer. Each run was

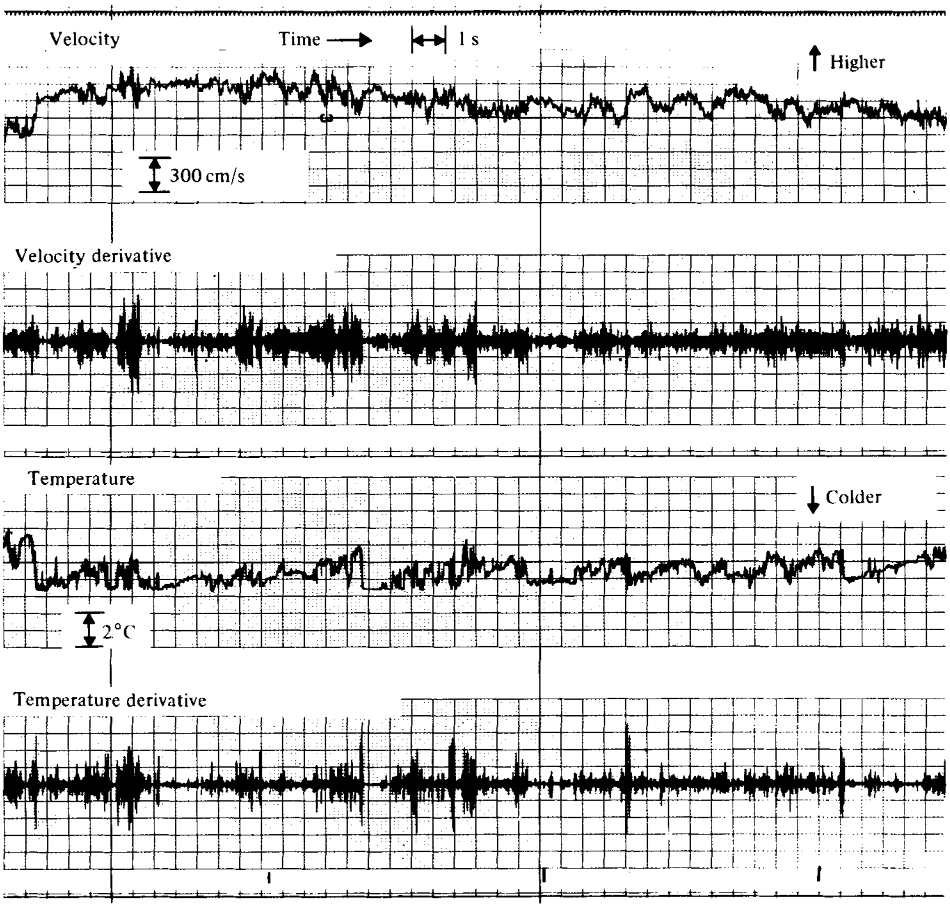

FIGURE 1. Typical strip-chart recording of turbulent temperature and velocity and their derivatives under unstable conditions.

divided into subsections of 8192 samples. The Fourier coefficients and spectral estimates of the time series in each subsection were computed. The estimates from all of the subsections were then averaged. Further smoothing was performed by averaging spectral estimates within 24 non-overlapping frequency bands, each band an equal interval on a logarithmic scale. The standard deviation of each averaged spectral estimate was used to calculate the 95% confidence intervals assuming a chi-squared probability distribution. Taylor's hypothesis was applied to convert the frequency spectra to wavenumber spectra.

Figures 2 and 3 show spectra of velocity and temperature derivatives (commonly referred to as dissipation spectra) for run RY25C. The spectra are very smooth to 2000 Hz. A region with slope ($\frac{1}{3}$) consistent with the prediction for the inertial subrange is clearly evident in both spectra. However, the temperature-derivative spectrum shows a marked increase in slope as the peak is approached. It should also be noted that the peak of the temperature-derivative spectrum is at a higher frequency than the peak of the velocity-derivative spectrum, i.e. the scales at which

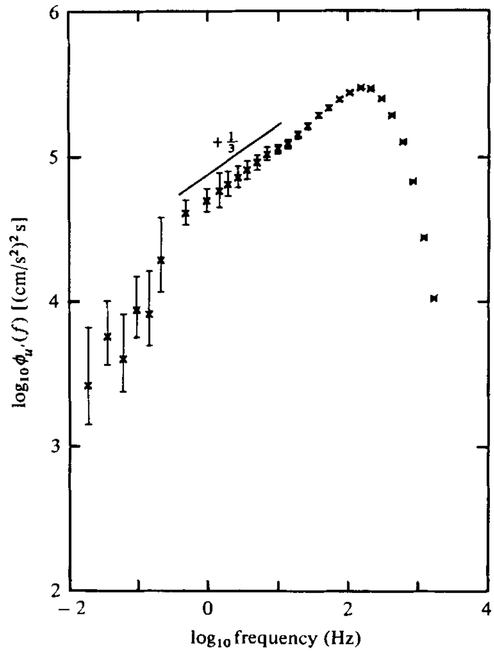

FIGURE 2. Typical spectrum of wind velocity derivative, run RY25C (with 95 % confidence intervals).

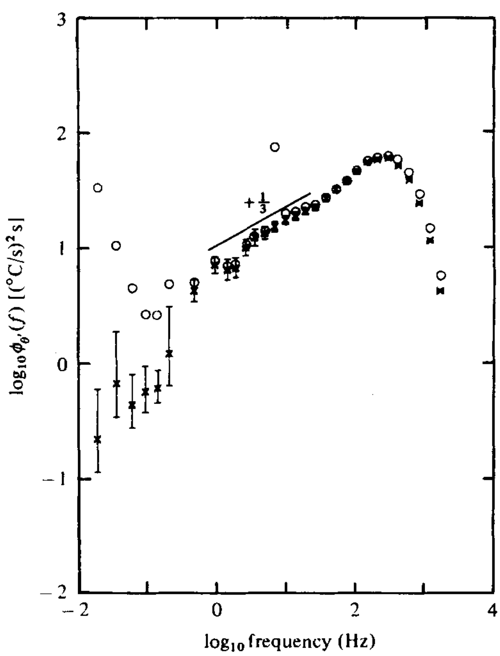

FIGURE 3. Typical spectrum of air temperature derivative, run RY25C (with 95 % confidence intervals). ○, uncorrected; ×, corrected.

Run	$\bar{u}$ (cm/s)	Re $\times 10^{-3}$	Re_λ	z/L	ϵ (cm²/s³)	α (10 Hz)	α (ISR)	ϵ_θ (°C²/s)	ϵ_θ noise correction (%)	β (10 Hz)	β (ISR)	Record length (s)	Plot symbol
RY21C	329	425	1780	+0·04	328	0·556	0·565	0·00569	34	0·90	0·97	237·6	+
RY22A	452	558	2600	−0·10	538	0·516	0·539	0·0876	2	0·94	0·95	233·5	✚
RY23B	490	601	3960	−0·08	804	0·553	0·557	0·125	2	1·06	1·00	237·6	⊠
RY24B	537	655	4280	−0·06	1136	0·545	0·542	0·133	2	1·11	1·07	235·6	⊠
RY25C	590	715	4150	−0·04	1335	0·536	0·537	0·0952	2	1·14	1·10	237·6	⧗
RY26B	602	734	3280	−0·04	1475	0·530	0·532	0·0772	3	1·10	1·10	237·6	×
RY27A	592	722	4170	−0·03	1423	0·513	0·527	0·0414	3	1·02	1·02	75·8	╳
B	575	706	2250	−0·02	1342	0·524	0·554	0·0299	4	1·02	1·01	77·8	|
RY28A	391	480	2130	−0·04	413	0·510	0·571	0·0137	15	0·92	0·89	77·8	
B	345	423	1310	−0·02	493	0·537	0·571	0·0030	49	0·97	1·00	77·8	
RY29A	344	422	1270	−0·01	364	0·464	0·494	—	—	—	—	77·8	
B	324	398	1200	−0·01	298	0·499	0·457	—	—	—	—	77·8	
C	296	365	1410	+0·02	143	0·535	0·533	$6{\cdot}7\times10^{-4}$	86	0·87	0·91	77·8	—
			Average			0·524	0·537			1·00	1·00		
			Standard deviation			0·025	0·010			0·09	0·03		
			Standard error of mean			0·007	0·003			0·03	0·01		

TABLE 1. Summary of the one-dimensional Kolmogorov constants for velocity and temperature together with conditions for each run. The mean wind speed is designated as $\bar{u}$, Re is the Reynolds number computed using $\bar{u}$ and z, the height of the measurements above ground (2 m). The turbulence Reynolds number is designated as Re_λ and is computed using the r.m.s. turbulent velocity $(\overline{u^2})^{\frac{1}{2}}$ and the Taylor microscale λ. L is the Monin–Oboukhov length. The noise correction for ϵ_θ is the percentage correction applied to the uncorrected spectra. Symbols are those used in figures showing normalized spectra. ISR = inertial subrange defined from k/k_s = 0·0013 to 0·013.

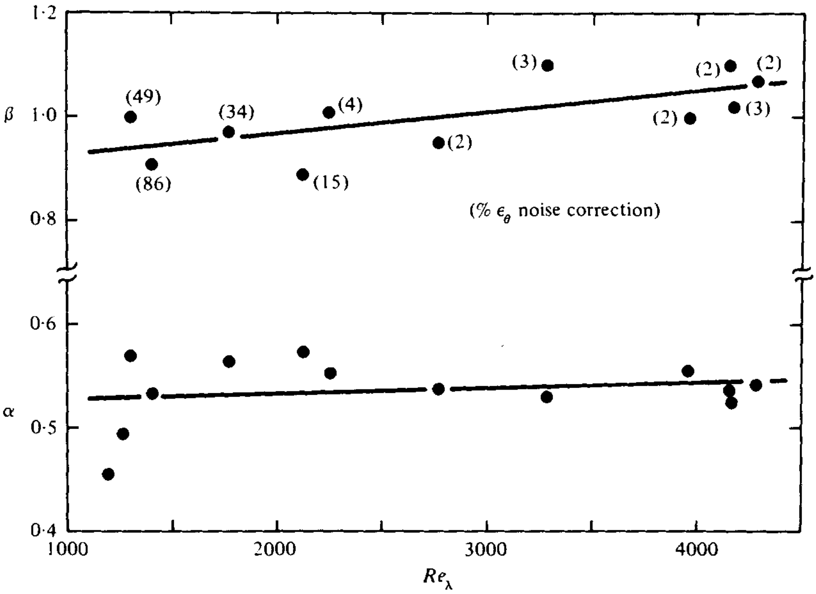

FIGURE 4. The variation of the Kolmogorov constants α and β with turbulence Reynolds number.

the dissipation of temperature fluctuations is a maximum are smaller than the scales associated with the maximum velocity dissipation.

The dissipation spectral estimates at the lowest six frequencies in figures 2 and 3 were obtained from the spectral estimates by use of the relation

$$\phi_{\partial x/\partial t}(f) = (2\pi f)^2 \phi_x(f), \tag{18}$$

where x represents either u or θ. Replacement of the low-frequency estimates obtained directly from the differentiated signals was necessary because of low-frequency noise introduced by the differentiating circuits. A correction was also made in the spectra of the differentiated temperature and velocity signals for high-frequency noise. This correction was made by subtracting from the uncorrected spectra the spectra computed during 'quiet' periods, i.e. periods when the dissipation was much less than the mean. Table 1 indicates the extent of the noise contribution to ϵ_θ before correction.

The spectral estimates have also been corrected for various other factors including aliasing, anti-alias filter response, differentiator deviation from the ideal, spatial averaging by the sensor and deviations from Taylor's hypothesis. The combined effect of the first three factors was negligible except at the highest three frequencies and hence had a negligible ($< 1\%$) effect on ϵ, ϵ_θ, α and β.

The corrections for spatial averaging due to finite sensor length for both velocity and temperature were based on the work of Wyngaard (1968, 1971*a*). For both velocity and temperature, the sensor length was about twice the microscale. This resulted in significant spectral underestimates over about the highest decade of frequency with the correction increasing with increasing frequency to a typical maximum of 20%.

The error in the spectral estimates due to fluctuating convection velocities causing

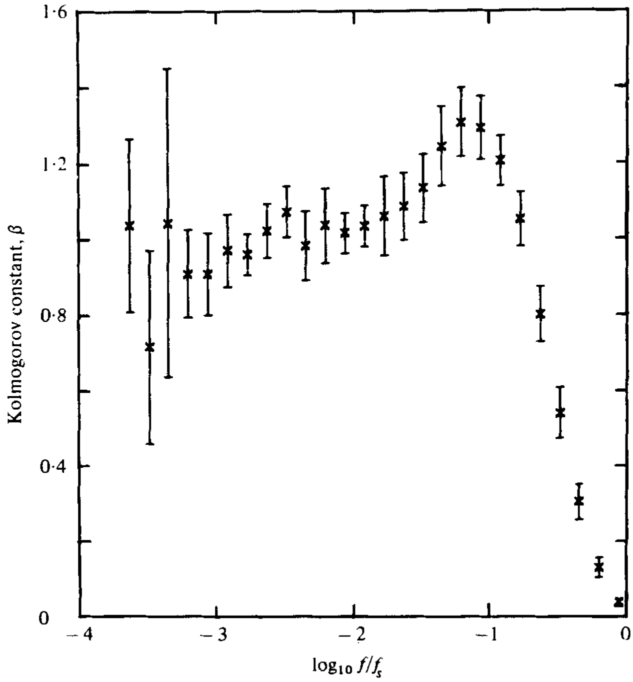

FIGURE 5. Variation of one-dimensional Kolmogorov constant for temperature as a function of normalized frequency (with 95 % confidence intervals). $\phi_\theta = \beta\epsilon^{-\frac{1}{3}}\epsilon_\theta k^{-\frac{5}{3}}$.

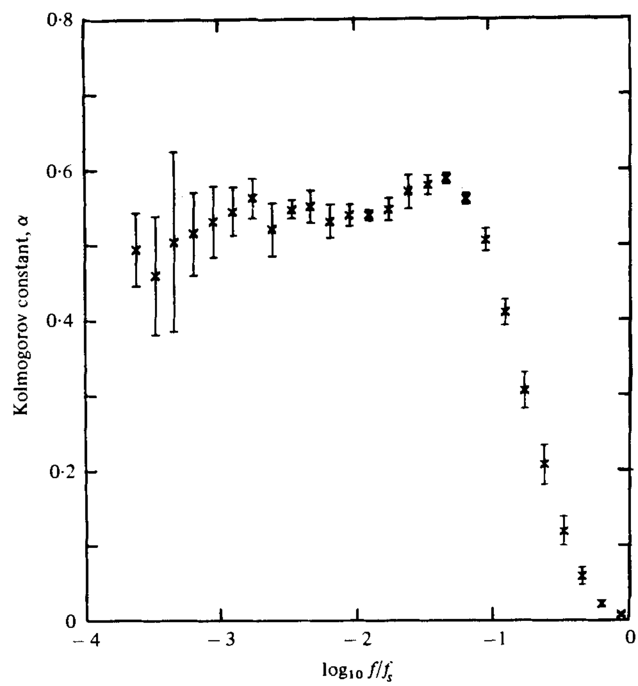

FIGURE 6. Variation of one-dimensional Kolmogorov constant for velocity as a function of normalized frequency (with 95 % confidence intervals). $\phi_u = \alpha\epsilon^{\frac{2}{3}}k^{-\frac{5}{3}}$.

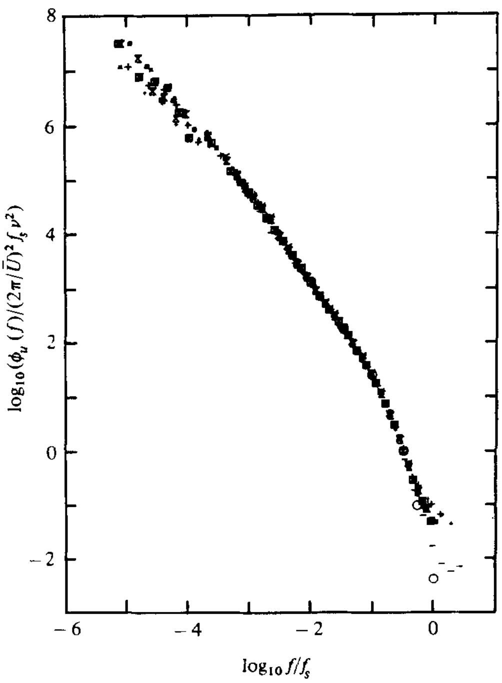

FIGURE 7. Composite normalized velocity spectra compared with Nasmyth (1970, circles).

inaccuracy in Taylor's hypothesis was estimated from a model developed by Lumley (1965). For the turbulence intensities encountered (~ 20 %) the spectral estimates were too large at frequencies below the dissipation peak (by about 3 %) and at higher frequencies (by as much as 50 %).

The effects of these individual corrections on the rates of dissipation and Kolmogorov constants were less than 15 % and when combined they tended to offset one another, so that the overall spectral correction was about 4 % for the inertial-subrange spectral levels, − 13 % for the mean kinetic energy dissipation rate, − 3 % for the mean thermal dissipation rate, + 5 % for α and − 2 % for β. An example of the magnitude of the corrections can be seen in figure 3.

Estimates of ϵ and ϵ_θ were obtained by integrating spectra of the derivative signals [(4) and (9)]. These values, together with the computed velocity and temperature spectra in the $-\frac{5}{3}$ range, enable calculation of the universal constants α and β for each run [(1) and (7)]. A summary of the estimated values is given in table 1 together with the conditions for each run. All averages were computed over the length of the run. The Monin–Oboukhov length L is defined by

$$L = (-\overline{uw})^{\frac{3}{2}}\, T/\kappa g\, \overline{w\theta}), \tag{19}$$

where w is the vertical velocity component, T the mean absolute temperature, κ von Kármán's constant (0·4) and g the gravitational acceleration.

The variation of β and α with the turbulence Reynolds number Re_λ is shown in

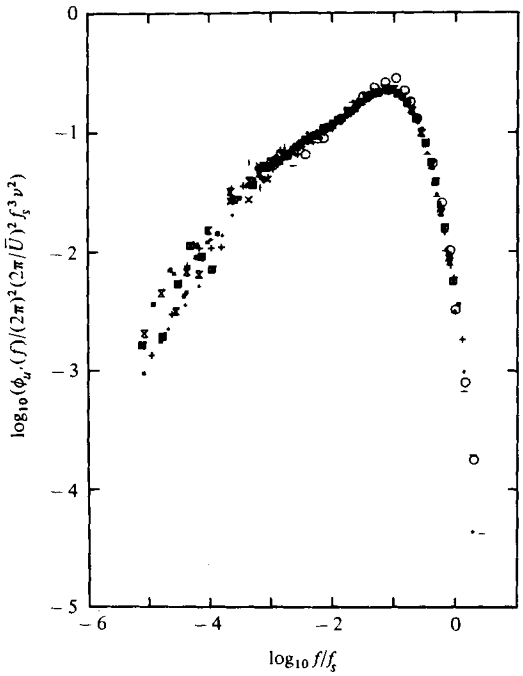

FIGURE 8. Composite normalized velocity-derivative spectra (velocity dissipation spectra) compared with Boston (1970, run 202(2) A, circles).

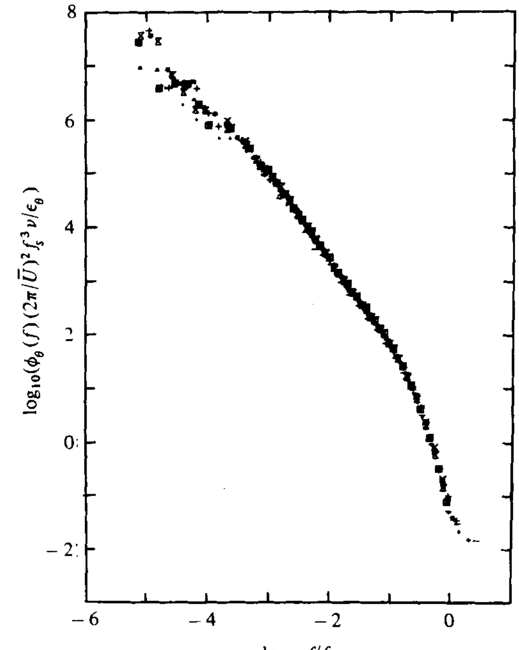

FIGURE 9. Composite normalized temperature spectra.

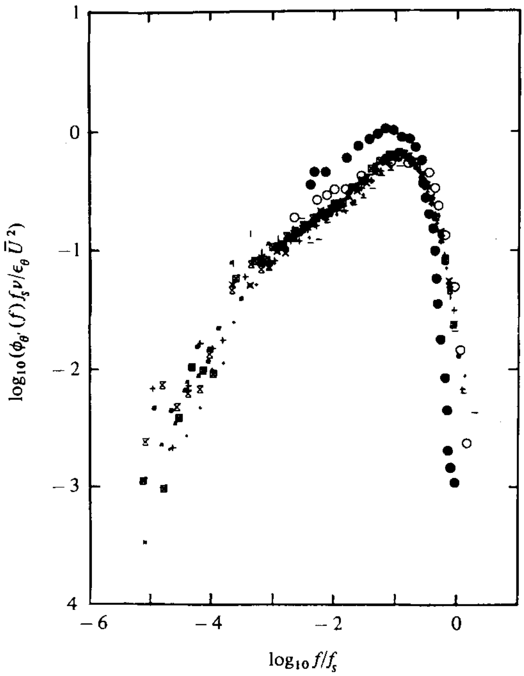

FIGURE 10. Composite normalized temperature-derivative spectra (temperature dissipation spectra) compared with Boston & Burling (1972, run 202(1) D, open circles) and Gibson, Stegen & Williams (1970, filled circles).

figure 4. Re_λ is equal to $\lambda(\overline{u^2})^{\frac{1}{2}}/\nu$, where λ is the Taylor microscale, defined by

$$\lambda^2 = \overline{u^2}/\overline{(\partial u/\partial x)^2}.$$

There is no significant variation in α. However there is an indication that β increases with Re_λ, suggesting that the turbulence may not have been locally isotropic, at least for $Re_\lambda < 3000$. The evidence for the variation in β is uncertain because of the large corrections for noise applied to several of the runs at the lower values of Re_λ. However the correction is based on subtracting noise spectra computed during quiet periods from the uncorrected spectra, which is likely to result in an anomalously high estimate of β.

The increase in slope of the temperature spectra as the frequency of maximum dissipation is approached is more clearly seen in figure 5, which is a plot of the Kolmogorov constant β *vs.* frequency normalized by the Kolmogorov microscale frequency f_s, given by

$$f_s = \frac{\overline{u}}{2\pi}(\epsilon/\nu^3)^{\frac{1}{4}} = \frac{\overline{u}}{2\pi}k_s. \tag{20}$$

Figure 6 shows a similar plot of α *vs.* normalized frequency which reveals a much smaller and perhaps insignificant effect. Changes in slope of the type shown in figure 5 were not discernible in the results of Boston & Burling (1972) or Gibson, Stegen & Williams (1970).

Figures 7–10 show the spectra from 9 runs normalized according to (5) and (11). These data do appear to verify the existence of universal spectral functions for both

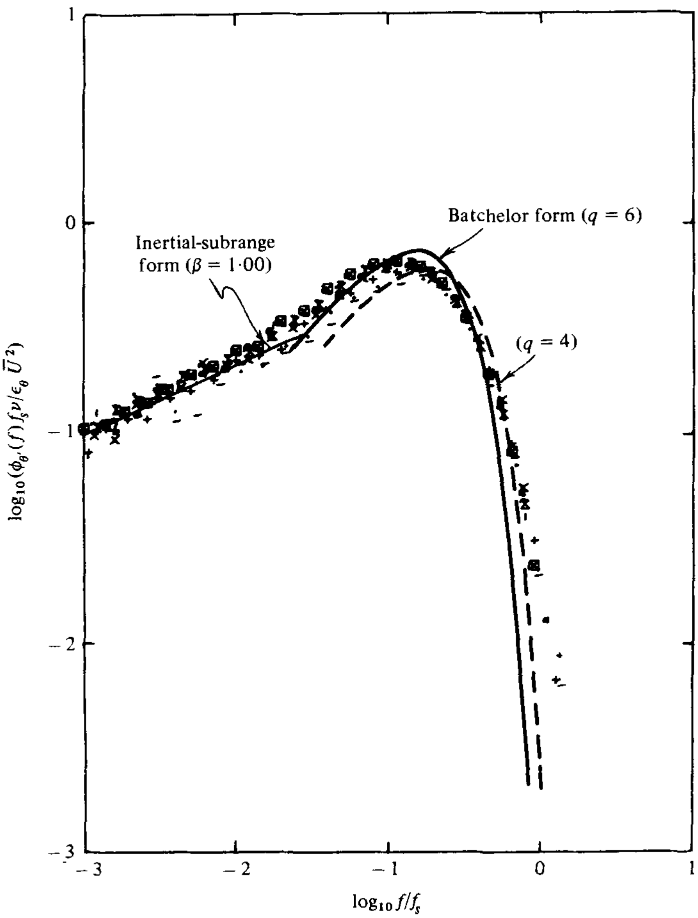

FIGURE 11. Comparison of Batchelor's (1959) prediction with observed spectra of the derivative of temperature.

velocity and temperature. Included on these plots are the results of Nasmyth (1970), Boston (1970), Gibson, Stegen & Williams (1970) and Boston & Burling (1972).

Even though the Prandtl number of air is 0·7, it may be useful to compare Batchelor's prediction (13) with observations because of the qualitative similarity between them (i.e. an increase in slope near the end of an inertial subrange) and because the arguments underlying the theory may be valid outside the range for which they were originally intended. A comparison with the observed dissipation spectra is shown in figure 11 for $q = 4$ and 6. The lower value of q results in fair agreement with the observations at the higher wavenumbers while the curve with $q = 6$ agrees better at lower wavenumbers. The value of k_*/k_s for $q = 6$ is 0·023, determined by matching (7) with (14) and taking $\beta = 1{\cdot}00$. This value of k_*/k_s is in good agreement with the observed break in the slope (approximately 0·02). The relation between β, q and k_*/k_s given in (15) is valid only if there is a range in which the dissipation spectrum has a slope of approximately one. In figure 11, the exponential cut-off in Batchelor's prediction occurs at wavenumbers low enough to prevent a slope of one.

5. Discussion

There is good agreement between observed velocity spectra (figures 7 and 8) and the suggested universal forms (5) and (6). There is remarkably little scatter in the non-dimensional spectra plotted in figure 7, particularly when one recalls that each plotted point is an independent spectral estimate. The agreement of the observations in figure 7 with Nasmyth's (1970) observations is excellent except at the highest wave-numbers, where the difference may arise because no corrections were made for noise in our velocity spectra. The agreement of the observed dissipation spectra (corrected for noise) with that of Boston (1970) is good except at the maximum of the spectrum, where Boston's observations are more peaked.

The value of the universal constant α, defined by (1)–(3), was determined to be $0{\cdot}54 \pm 0{\cdot}01$ (averaged over $0{\cdot}0013 \leqslant k/k_s \leqslant 0{\cdot}013$), which is in good agreement with several direct estimates. For example, Boston & Burling (1972) found 0·51 from atmospheric observations, Pond *et al.* (1966) summarized four investigations in the atmosphere, ocean and laboratory to find 0·48, Nasmyth (1970) found 0·56 from measurements in the ocean and Wyngaard & Coté (1971) found 0·50 from atmospheric measurements. In less good agreement are estimates of 0·65 reported by Kistler & Vrebalovich (1966) from laboratory observations of grid turbulence, 0·65 reported by Shieh, Tennekes & Lumley (1971) from atmospheric measurements and 0·69 reported by Gibson, Stegen & Williams (1970) also from atmospheric measurements. As discussed in Schedvin, Stegen & Gibson (1974), at least some of these estimates may be too high owing to the effect of attenuation of the small scales caused by finite sensor length.

Although the scatter is greater than for velocity spectra, the non-dimensional plots (figures 9 and 10) of temperature spectra appear to satisfy the predictions [(11) and (12)] of universal similarity reasonably well. The scatter may be partly related to the higher degree of intermittency in the turbulent temperature fluctuations than in the corresponding fluctuations of velocity. Part of the scatter might also be accounted for by the evidence shown in figure 4 that the Reynolds number might not have been sufficiently high for the existence of an inertial subrange. This possibility is particularly strong for runs with a turbulence Reynolds number below 2000. There is an indication in figure 10 that spectra from runs RY21C and RY29C fall systematically below the mean of all of the observations. These are the only two runs with stable stratification. Kaimal *et al.* (1972) report that a ratio of $\frac{4}{3}$ between the transverse and longitudinal spectral levels is observed at wavelengths of order equal to or less than the height z above the ground under unstable stratification but at wavelengths of order equal to or less than $\frac{1}{10}z$ under stable conditions. Hence conditions for local isotropy may not have been fulfilled for a substantial part of the low-frequency end of the two stable runs. There is, however, no evidence of a departure of the velocity spectra from the universal form for these runs.

Figure 10 shows poor agreement with the results of Gibson, Stegen & Williams (1970) and fair agreement with Boston & Burling (1972). The disagreement may in part be explained by contamination of the observations of Gibson, Stegen & Williams and Boston & Burling by noise. The noise corrections made by Boston & Burling were typically much larger than those required in our case. The observations of

Gibson, Stegen & Williams were made over the sea, where the amplitude of turbulent temperature fluctuations was approximately an order of magnitude smaller than the largest amplitudes which we observed over land. Schmitt, Friehe & Gibson (1977) attribute spectral variations in the Gibson, Stegen & Williams (1970) data to sensitivity of the temperature sensor to humidity fluctuations. Recent observations of temperature spectra over land by Champagne *et al.* (1977) are in good agreement with our observations in figures 9 and 10. The Champagne *et al.* observations show a change in slope consistent with the observations reported here.

The mean value of β estimated from our observations is 1·00, which is in poor agreement with Gibson, Stegen & Williams (1970), who found a value of 2·3, and with Boston & Burling (1972), who found $\beta = 1{\cdot}6$. There is however fair agreement with the direct estimate by Champagne *et al.* (1977) of $\beta = 0{\cdot}82$. There is also fair agreement with the indirect estimates summarized by Monin & Yaglom (1975, p. 504), who suggest that $\beta = 0{\cdot}75$. Many of the indirect estimates of β are based on approximating the conservation equation for the variance of turbulent temperature fluctuations by neglecting the divergence of the vertical transport of $\overline{\theta^2}$, i.e. by assuming that local production equals local dissipation. Observations reported by Wyngaard & Coté (1971) suggest that there is a local excess of export over import of the transport of $\overline{\theta^2}$ of about 10 % of the production at $z/L = -0{\cdot}1$. Correction of the indirect estimates of β for neglect of the transport term would therefore reduce the difference between the indirect estimates and the direct estimate reported here.

The variation of β with turbulence Reynolds number shown in figure 4 is puzzling, particularly because there is no significant variation in α. If the turbulent velocity fluctuations are locally isotropic, it is difficult to understand how the temperature fluctuations could be anisotropic. It is possible that the velocity fluctuations were not locally isotropic in the $+\frac{1}{3}$-range, but that the estimate of α was not strongly affected. Pond *et al.* (1966) find little variation in α over a wide range of Reynolds numbers even though the ratios of streamwise and cross-stream velocity spectra in the $-\frac{5}{3}$-range did not always fulfil the requirement for isotropy. The skewness of the temporal temperature derivatives averaged to $-0{\cdot}8$, indicating that a departure from isotropy may be a factor in explaining the variation of β with Re_λ. The variation shown in figure 4 agrees with an estimate of $\beta = 0{\cdot}7$ determined by Gibson & Schwarz (1963) from laboratory measurements in which Re_λ was about 100. Figure 4 suggests that the limiting or correct value of β is at least as large as 1·05 (the average value of β for $Re_\lambda > 3000$). Clay (1973) summarizes measurements of β and its variation with Reynolds number which also show (on average) that β increases with Re.

The observed increase in slope with increasing wavenumber of the temperature spectra (figures 5 and 11) beginning at normalized wavenumbers of about 0·02 was unexpected and puzzling. Previous observations in the atmosphere (Boston & Burling 1972; Gibson, Stegen & Williams 1970) do not show this characteristic. It was first thought that the change in slope was due to errors in the measurements or analysis. However, a search for sources of errors discounted several possibilities and the validity of the results was further supported by measurements of Champagne *et al.* (1977), which showed a similar change in slope. A plausible qualitative explanation is the suggestion by Schmitz (1968) that spatial dissimilarity in the dissipation of temperature and velocity fluctuations would result in a change in slope of the temperature spectrum in fluids with Pr of order one qualitatively similar to that predicted by Batchelor (1959). The explanation for the increase in slope of the temperature spectrum of fluids with $Pr \gg 1$ is that, for scales smaller than the Kolmogorov microscale, all of the velocity fluctuations have dissipated, while temperature fluctuations remain which are transferred to smaller and smaller scales by the larger-scale deformation of the fluid until the gradients become so large that they are dissipated by molecular diffusion. In the case of fluids with Pr of order one, it is hypothesized that some regions having large small-scale temperature fluctuations are not associated with similar velocity fluctuations. These regions must rely on slower large-scale deformation to increase the temperature gradients until dissipation occurs.

Spatial dissimilarities in the small-scale structure of the flow may be caused by dissimilarities in the production and transport of turbulent velocity and temperature fluctuations on larger scales. The terms in the budget equations for the velocity and temperature variance (see e.g. Wyngaard & Coté 1971) are not similar. For example, pressure forces play a role in the transport of momentum, while there is no corresponding mechanism for temperature. As a second example, consider an unstably stratified atmospheric shear layer near the ground. It is observed that the vertical gradient of mean temperature decreases faster with height than the gradient of mean velocity and may approach zero for sufficiently large heights (see, for example, Businger *et al.* 1971). Therefore the production of turbulent temperature fluctuations, given by

$$\overline{\theta w}\,\partial\Theta/\partial z,$$

where w is the vertical turbulent velocity component and Θ is the mean potential temperature, decreases more rapidly with increasing height than the production of velocity fluctuations, given by

$$\overline{uw}\frac{\partial U}{\partial z}+\frac{g}{\Theta}\,\overline{\theta w},$$

where U is the mean velocity. Assuming that divergence of vertical transport of temperature and velocity fluctuations is small compared with production, one would expect large-scale descending turbulent eddies observed at a fixed height to have an excess of small-scale velocity fluctuations over temperature fluctuations when compared with ascending eddies. This expectation is consistent with the observed higher degree of intermittency (quiet periods, larger kurtosis) in the small-scale temperature structure when compared with the small-scale velocity structure (figure 1).

There is fair agreement shown in figure 11 between the theory of Batchelor (1959) and the observed temperature spectra even though the theory was intended for flows with larger Prandtl number. The non-dimensional wavenumber at which the break in slope of the temperature spectra occurs is predicted by Batchelor to be a universal constant k_*/k_s of order one. This may be compared with the results of this paper and oceanic observations by Grant *et al.* (1968), which show that $k_*/k_s = 0{\cdot}02$. Grant *et al.* estimated that q is approximately equal to four, but they did not determine ϵ_θ directly. If ones takes $\beta = 1{\cdot}0$ and $k_*/k_s = 0{\cdot}02$, (15) yields $q = 6$. This may be compared with Batchelor's (1959) suggestion that q is equal to about two and observations in the laboratory by Gibson & Schwarz (1963) yielding the same value. Gibson (1968) suggested on theoretical grounds that q is about 2 with lower and upper bounds of $\sqrt{3}$ and $2\sqrt{3}$ and that the corresponding value of β lies in $1{\cdot}4-0{\cdot}1 \leqslant \beta \leqslant 1{\cdot}4+0{\cdot}6$. However, Gibson's arguments do not require k_*/k_s to be a constant and indeed these estimates of q and β yield [by (15)] $0{\cdot}21-0{\cdot}05 \leqslant k_*/k_s \leqslant 0{\cdot}21+0{\cdot}02$, which is about an order of magnitude larger than observations. Gibson & Schwarz found k_*/k_s to be equal to about 0·1 as did Clay (1973), also from laboratory measurements in water. Gibson, Lyon & Hirschsohn (1970) found k_*/k_s to be approximately $\frac{1}{30}$ from laboratory measurements of conductivity fluctuations in water solutions downstream from a sphere. These results suggest that the value of k_*/k_s may depend on the Reynolds number. This suggestion is consistent with the observation that high-frequency turbulent temperature fluctuations become more intermittent with increasing Re and therefore that the spatial dissimilarity in the dissipation of temperature and velocity fluctuations may occur at lower normalized wavenumbers.

Examination of figure 11 shows that the slope of the temperature dissipation spectrum changes from $\frac{1}{3}$ to about $\frac{1}{2}$ over less than a decade of wavenumbers. This change corresponds to the dissipation correlation coefficient in Van Atta's (1971, 1973) model (17) changing from $\frac{2}{3}$ to $-\frac{1}{3}$. It is physically plausible that $\rho(r)$ may decrease with decreasing r. Indeed, Antonia & Van Atta's (1975) observations in the laboratory ($Re_\lambda = 240$) show ρ decreasing from 0·8 to 0·5 as rk_s decreases from 3000 to 6. However, a large change from positive to negative values of ρ over a small range of r seems unlikely.

The preceding discussion has pointed out progress in understanding turbulent microscale processes; however it has also revealed much of our ignorance. Many questions remain to be answered concerning the effects of Reynolds number, stability and anisotropy on the small-scale structure of turbulent flows.

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